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# Mathematical Modelling of Traffic Flow Dynamics Around Roundabouts Using Navier–Stokes and Advection–Diffusion Equations

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***Abstract:*** **Traffic congestion is a persistent challenge in urban transport systems, particularly around roundabouts where nonlinear merging and circulating flows create instability and delays. Traditional microscopic, mesoscopic, and macroscopic models often neglect explicit treatment of priority rules, capacity drops, and queue spillback, limiting their ability to predict roundabout-induced congestion. To address this gap, this study develops a coupled Navier–Stokes–advection–diffusion framework for simulating traffic dynamics at roundabouts. The governing equations are discretized using the finite volume method with a QUICK scheme for spatial accuracy and advanced in time with a Crank–Nicolson integration for stability. Non-dimensionalization ensures general applicability across different traffic environments. Simulation results reproduce fundamental traffic relations, identify optimal densities for maximum throughput, and reveal oscillatory stop-and-go waves consistent with empirical observations. Scenario analysis shows that increased diffusion enhances stability but reduces flow capacity, capturing real-world trade-offs between efficiency and control. The study concludes that embedding yield laws and circulating feedback into continuum equations provides a rigorous foundation for analyzing roundabout performance. Policy recommendations include adopting geometry- and flow-calibrated entry controls, while future research should incorporate stochastic demand and adaptive control strategies to refine predictive accuracy. This contribution provides both theoretical innovation and practical insights for sustainable traffic management.**

***Keywords:*** **Advection–diffusion modelling; Congestion management; Navier–Stokes equations; PDE-based simulation; Roundabout performance; Traffic flow dynamics.**

## I. INTRODUCTION

### A. Background Information

Traffic congestion is a persistent global challenge with profound economic, social, and environmental implications [1, 2]. In many metropolitan areas, the scope for further road expansion has been exhausted, making efficient traffic management strategies essential [3]. Rising vehicle numbers intensify the problem: in the United States alone, travel delays and the Road Congestion Index have increased by 20% over the past decade, with annual economic losses exceeding $100 billion due to lost productivity, wasted fuel, and environmental costs [4]. Similar pressures are evident in Europe and Asia, where congestion continues to erode quality of life and urban efficiency.

Mathematical modelling offers a rigorous framework for analyzing and predicting traffic flows [5]. Existing models include microscopic car-following approaches, mesoscopic and kinetic formulations inspired by statistical mechanics, and macroscopic continuum models [6]. Of particular relevance are macroscopic formulations, where the Navier–Stokes equations are adapted to capture aggregate flow characteristics, while advection–diffusion equations describe the dispersion of disturbances in space and time [3, 7]. These approaches have successfully explained shock waves, queue formation, and self-organization, contributing to the development of intelligent transportation systems. More recent studies have introduced probabilistic methods to account for heterogeneous arrivals and complex flow dynamics [8].

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A significant gap remains in understanding traffic behavior at roundabouts. Unlike signalized intersections, roundabouts redistribute traffic in nonlinear ways [9]: while they often enhance flow capacity and safety [10], they can also generate turbulence [11], queue spillback [12, 13], and localized bottlenecks [14] if poorly designed. Empirical studies confirm these dual outcomes, yet theoretical models that explicitly examine roundabout-induced dynamics remain limited. Most macroscopic and numerical studies continue to assume uninterrupted flow conditions and rarely extend their analysis to intersection geometries [15]. This gap risks underestimating the influence of roundabouts on traffic stability, efficiency, and safety, especially in congested urban contexts where travel demand is rapidly increasing.

Accordingly, we advance a rigorous fluid–dispersion framework that couples a Navier–Stokes–type momentum balance with an advection–diffusion transport law to quantify the influence of roundabouts on traffic flow. The model resolves upstream–entry–circulation–exit dynamics, enabling calibrated assessment of continuity (queue formation/dissipation), capacity (entry and circulating throughput), and stability (disturbance amplification/decay). The resulting evidence informs intersection geometry and control, guides urban traffic policy, and supports sustainable congestion management across dense networks.

**B. Contribution**

This study advances traffic flow modelling by integrating a Navier–Stokes–type momentum balance with an advection–diffusion transport equation explicitly tailored to roundabouts. Unlike existing macroscopic and kinetic models, which often treat intersections statistically or neglect priority rules, the proposed framework embeds yield laws and circulating–entry interactions directly as boundary/interface conditions. This formulation allows for the explicit computation of capacity drop, spillback risk, and stability margins within a continuum PDE system. By employing high-order spatial discretization (QUICK scheme), temporally stable Crank–Nicolson integration, and non-dimensional scaling, the study introduces a rigorous numerical framework capable of capturing stop-and-go oscillations, density–speed–flow trade-offs, and throughput maxima without resorting to empirical curve-fitting. The contribution lies in bridging theory, simulation, and policy by demonstrating how roundabout-specific priority dynamics can be quantified mechanistically, providing new predictive insights for geometry design and control policy evaluation.

## II. RELATED WORKS

### A. Theoretical Formulation

We model traffic on a two–dimensional road domain $\Omega \subset \mathbb{R}^2$ that contains a roundabout region $\Omega_{\mathrm{rb}} \subset \Omega$. The state variables are the *traffic density* $\rho(x,y,t)$ (vehicles per unit area) and the *mean velocity* $\mathbf{u}(x,y,t) = (u,v)$ (m s $^{-1}$). The model couples: (i) conservation of vehicles, (ii) a Navier–Stokes–type momentum balance with a *traffic pressure* $p(\rho)$, and (iii) an advection–diffusion closure that captures small-scale mixing (lane changes, reaction variability).

**1) Governing Equations**

**Mass conservation (continuity)**

$$\partial_t \rho + \nabla \cdot (\rho \ \mathbf{u}) = 0, \qquad (1)$$

stating that the local rate of change of density plus the divergence of the vehicle flux $\rho \ \mathbf{u}$ vanishes.

**Momentum balance (Navier–Stokes analogue).**

$$\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{1}{\rho} \nabla p(\rho) + \nu \ \nabla^2 \mathbf{u} + \mathbf{F}(x,y,t). \qquad (2)$$

The left–hand side is material acceleration. On the right, $-(1/\rho)\nabla p(\rho)$ models repulsive car–car interactions (drivers increase headway as density rises), $\nu \nabla^2 \mathbf{u}$ is an *effective viscosity* (relaxation/heterogeneity), and $\mathbf{F}$ aggregates localized forcings (e.g., control, incidents, grades).

**Density dispersion (advection–diffusion).**

$$\partial_t \rho + \nabla \cdot (\rho \ \mathbf{u}) = D \ \nabla^2 \rho + S(x,y,t). \qquad (3)$$

where $D \geq 0$ is a *mixing* coefficient smoothing sharp density gradients; $S$ is a source/sink for entries/exits (positive for inflow, negative for outflow). In practice, (3) augments (1) to represent sub-grid dispersion.

**2) Constitutive Closures and Forcing**

**Traffic pressure $p(\rho)$.**

A monotone law $p'(\rho) > 0$ encodes compressibility: larger $\rho$ induces stronger opposition to compression. Typical choices include smooth surrogates of a fundamental diagram; e.g.,

$$p(\rho) = \alpha \frac{\rho}{1-\rho/\rho_{\max}}, \qquad \alpha > 0,\ 0 < \rho < \rho_{\max},$$

where $\rho_{\max}$ is jam density.

**Effective viscosity $\nu$ and density diffusion $D$.**

Constants $(\nu, D)$ provide baseline damping; density-dependent forms $(\nu(\rho), D(\rho))$ can reflect increased heterogeneity near congestion.

**External forces $\mathbf{F} = \mathbf{F}_{\mathrm{ctrl}} + \mathbf{F}_{\mathrm{obs}}$.**

$\mathbf{F}_{\mathrm{ctrl}}$ represents soft controls (advisory speed, lane guidance). Incidents/obstacles are modeled with a localized repulsion:

$$\mathbf{F}_{\mathrm{obs}}(x) = k_{\mathrm{obs}} \frac{x-x_0}{\|x-x_0\|} \chi_{B_R(x_0)}(x), \tag{4}$$

with center $x_0$, radius $R$, strength $k_{\mathrm{obs}} > 0$, and indicator $\chi_{B_R}$.

**3) Roundabout Interfaces and Boundary Conditions**

Let $\Gamma_{\mathrm{in}}$ be approach entries, $\Gamma_{\mathrm{out}}$ the exits, and $\partial\Omega_{\mathrm{rb}}$ the give-way interface where entering streams yield to circulating flow. **Inflow (demand) on $\Gamma_{\mathrm{in}}$.** Prescribe normal flux

$$q_{\mathrm{in}}(t) = \rho\, u_n \quad \mathrm{on}\,\Gamma_{\mathrm{in}}, \qquad u_n := \mathbf{u}\cdot\mathbf{n}. \tag{5}$$

**Outflow on $\Gamma_{\mathrm{out}}$.** Use *convective* (do-nothing) outflow for $\mathbf{u}$ and homogeneous Neumann for $\rho$ to avoid artificial reflection. **Yield/priority on $\partial\Omega_{\mathrm{rb}}$.** Enforce a flux limiter reflecting gap acceptance:

$$\rho\, u_n \le \mathcal{F}_n(\rho_{\mathrm{circ}}) := \min\{\mathcal{C}_{\max}, \mathcal{C}_{\mathrm{gap}}(\rho_{\mathrm{circ}})\}, \qquad \mathcal{C}_{\mathrm{gap}}{}'(\cdot) < 0, \tag{6}$$

where $\rho_{\mathrm{circ}}$ is the circulating density near the merge; as circulating density rises, acceptable entry capacity drops.

**4) Combined System**

Equations (2) and (3) with the above closures and interfaces give the roundabout flow model:

$$\boxed{\begin{aligned} \partial_t\rho + \nabla\cdot(\rho\,\mathbf{u}) &= D\,\nabla^2\rho + S, \\ \partial_t\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} &= -\frac{1}{\rho}\nabla p(\rho) + \nu\,\nabla^2\mathbf{u} + \mathbf{F}_{\mathrm{ctrl}} + \mathbf{F}_{\mathrm{obs}}, \end{aligned}} \tag{7}$$

subject to (5)–(6) and appropriate initial conditions $\rho(x,y,0)$, $\mathbf{u}(x,y,0)$.

**5) One–Dimensional Reductions for Arms**

Along a single approach/exit parameterized by $x$ (tangent to flow), the model reduces to:

$$\partial_t\rho + \partial_x(\rho u) = D\,\partial_{xx}\rho + S(x,t), \qquad \partial_t u + u\,\partial_x u = -\frac{1}{\rho}\partial_x p(\rho) + \nu\,\partial_{xx}u + f(x,t), \tag{8}$$

with coupling at the roundabout via the flux limit (6). Here $u(x,t)$ is the tangential speed and $f$ a 1D projection of $\mathbf{F}$.

**6) Modelling Assumptions**

1. Vehicles are homogenized into a single class; $\rho, \mathbf{u}$ are mean fields (meso–macro scale).

2. Geometry is fixed during the analysis window; control and incidents enter via $\mathbf{F}$ and $S$.

3. $(p, \nu, D)$ are smooth and calibrated from data (e.g., speed–density curves, shock damping rates).

4. The mixing terms $\nu\nabla^2\mathbf{u}$ and $D\nabla^2\rho$ represent unresolved heterogeneity and lane mixing.

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**7) Symbols and Units**

**TABLE I: SUMMARY OF PARAMETERS USED IN THE STUDY**

| Symbol | Name |
|---|---|
| $\rho(x,y,t)$ | density (veh m $^{-2}$); $\rho_{\max}$ jam density |
| $\mathbf{u}=(u,v)$ | mean velocity (m s $^{-1}$); $u_n=\mathbf{u}\cdot\mathbf{n}$ normal component |
| $p(\rho)$ | traffic pressure (m $^{2}$ s $^{-2}$ up to scaling); $p'(\rho)>0$ |
| $\nu$ | effective viscosity (m $^{2}$ s $^{-1}$) |
| $D$ | density diffusion (m $^{2}$ s $^{-1}$) |
| $S(x,y,t)$ | source/sink (veh m $^{-2}$ s $^{-1}$) for entries/exits |
| $\mathbf{F}_{\mathrm{ctrl}}$, $\mathbf{F}_{\mathrm{obs}}$ | control and obstacle forces (m s $^{-2}$) |
| $\mathcal{F}_n, \mathcal{C}_{\max}, \mathcal{C}_{\mathrm{gap}}$ | entry flux capacity terms (veh m $^{-1}$ s $^{-1}$) |
| $\Gamma_{\mathrm{in}}, \Gamma_{\mathrm{out}}, \partial\Omega_{\mathrm{rb}}$ | entries, exits, yield interface |

$D$ damps steep density fronts (queue smoothing); $\nu$ damps velocity shear (stability); $p'(\rho)$ modulates shock formation (compressibility). The yield law (6) is the sole place where roundabout priority is enforced, enabling policy/design experiments (gap acceptance, metering) without altering the PDE core.

**B. Empirical Review**

Bellomo [6] synthesizes microscopic, kinetic, and macroscopic traffic theories, formalizing conservation laws and constitutive closures for flow–density–speed relations while emphasizing nonlinear wave phenomena and phase transitions. The prevailing view is that fluid–kinetic models capture aggregate dynamics and emergent shocks but idealize interfaces and controls. This motivates our focus on embedding geometry-specific laws for merges and yields into continuum PDEs to study roundabouts. A key critique is the lack of explicit treatment of intersection priority and capacity drops within hydrodynamic balances. The way forward is a hybrid Navier–Stokes–advection–diffusion framework coupled to interface flux constraints reflecting gap acceptance.

Shvetsov [3] surveys dynamic traffic assignment and macroscopic modelling on networks, organizing prediction, simulation, and optimization methods and highlighting the role of conservation-based PDEs for speed, delay, and queues. Current thinking treats geometry statistically while using continuum dynamics for control analysis. This motivates our research to couple network demand with local roundabout interface conditions in a single PDE setting. The critique is that yield-controlled entries and circulating feedback are not formulated as boundary/interface laws. The way forward is to encode priority via flux limiters driven by circulating density, enabling stability and capacity analysis without abandoning continuum structure.

Darbha [4] review macroscopic, microscopic, cellular automata, and kinetic approaches, linking modelling to ITS and quantifying congestion's economic burden while discussing string stability and information usage. Consensus holds that no single paradigm spans all scales, particularly near control elements. This motivates embedding decision rules in continuum balances to probe stability around merges and yields. The critique is limited representation of intersection geometries and capacity drop within rigorous PDE boundary conditions. The way forward is a Navier–Stokes–type momentum with advection–diffusion dispersion, augmented by data-driven interface capacities to assess throughput, shock damping, and policy levers at roundabouts.

Kiselev [7] propose a continuum model for controlled roads that captures self-organization, traveling jams, and the effects of traffic lights and speed humps via momentum equations constrained by speed/acceleration limits and driver response. Current approaches show how boundary forcing reproduces control-device impacts. This motivates our explicit interface formulation for roundabouts. A critique is scope: yield priority and circulating interactions are not modeled, and capacity drop is implicit rather than law-based. The way forward is to extend their controlled-road paradigm with flux limiters at yield lines and pressure/viscosity closures to evaluate stability and capacity under varying demands.

TFedotkin [8] develops a nonlocal probabilistic description for inhomogeneous flows with dependent arrivals, approximating grouped arrivals by nonordinary Poisson processes and queuing analogies. The current view accepts strong stochasticity in inflows and lane interactions. This motivates coupling stochastic sources to macroscopic PDEs at roundabout entries to represent bursty demand. The critique is a weak connection to continuum momentum/continuity and to geometric interfaces. The way forward is hybridization: drive the source term $S(x,y,t)$ and capacity distributions by their framework, then assess induced stability, delay, and spillback within a Navier–Stokes–advection–diffusion model with yield constraints.

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Song [15] advances macroscopic traffic models through scalar conservation laws, calibrated fundamental diagrams, multilane couplings, and high-resolution numerics validated on US-101 and I-80. The current approach tightly links data, closures, and solvers to recover shocks and rarefactions. This motivates applying the same rigor to roundabouts where merge–weave dynamics dominate. The critique is that yield-governed interfaces and circulating feedback are not posed as explicit PDE boundary laws. The way forward is to embed interface flux constraints and localized forces into Navier–Stokes–advection–diffusion balances, calibrating $p(\rho)$, $\nu$, $D$, and capacity–gap functions from approach/circulating data to evaluate stability and capacity.

### C. Critique and Research Gap

Current continuum traffic models capture large-scale patterns—such as waves, shocks, and phase changes—but they rarely describe how intersections actually work. In particular, they do not encode priority, gap acceptance, or capacity drop as explicit boundary/interface laws, a gap that is most visible at roundabouts [6, 3, 4]. Studies of controlled roads show that suitable boundary forcing can mimic signals and calming devices, yet they stop short of writing yield rules and circulating-flow feedback as precise interface conditions; capacity effects remain implicit rather than constitutive [7]. At the same time, evidence for bursty, dependent arrivals is strong, but stochastic inflow models are weakly linked to macroscopic conservation laws and are seldom projected onto geometry-aware source/sink terms or capacity distributions at merges and yield lines [8]. Even recent, data-calibrated macroscopic models with high-resolution numerics underperform at roundabouts because merge–weave interactions and yielding are not posed as PDE interface conditions [15].

These gaps motivate a clear way forward: (i) couple a Navier–Stokes–type momentum balance with an advection–diffusion law to represent density dispersion; (ii) introduce explicit, law-based interface constraints for roundabout entries using flux limiters driven by circulating density (to encode gap acceptance and capacity drop); and (iii) integrate stochastic demand through calibrated source/sink processes $S(x, y, t)$ while estimating $p(\rho)$, $\nu$, and $D$ from approach and circulating data [6, 3, 4, 7, 8, 15]. The need is not for entirely new PDEs, but for embedding roundabout-specific priority and feedback into these PDEs to quantify throughput, stability, spillback risk, and design–policy trade-offs.

## III. PROPOSED METHODOLOGY

This study develops a mathematical model to simulate traffic flow dynamics around roundabouts using a combination of the Navier–Stokes and advection–diffusion equations. The proposed methodology involves three main stages: discretization of governing equations, application of numerical schemes for space and time, and non-dimensionalization for generalization.

### A. Finite Volume Discretization

The governing equations are discretized using the finite volume method (FVM), which ensures local and global conservation of traffic density and momentum. A structured two-dimensional grid is adopted, with cell centers denoted by $(i, j)$ and face velocities represented by $(u, v)$ (Fig. 1). The grid spacing is set to $\Delta x = \Delta y = 1$. Cell-averaged density and velocity are denoted by $\rho_{i,j}$ and $\mathbf{v}_{i,j}$, respectively.

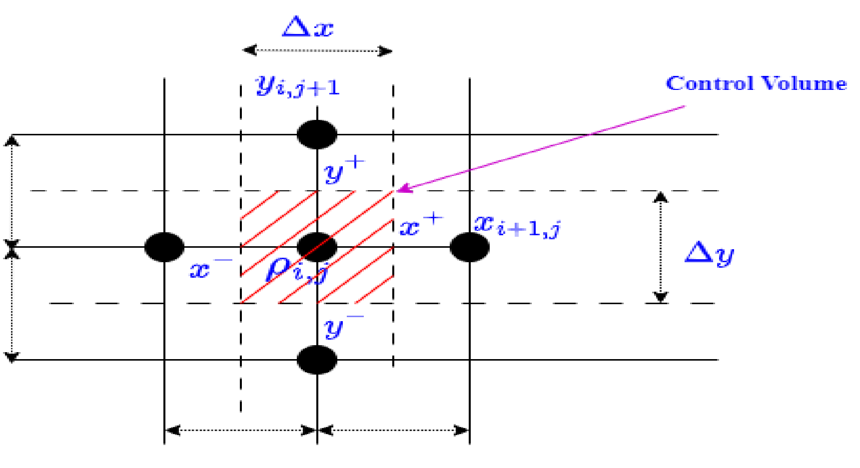


**Fig. 1: Control volume depicting the discretized grid system used in the finite volume formulation.**

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To enhance spatial accuracy, the Quadratic Upstream Interpolation for Convective Kinematics (QUICK) scheme is applied to the convective terms of both density and velocity equations. For the density equation, this yields:

$$\frac{\Delta\rho_{i,j}}{\Delta t}+\frac{1}{\Delta x}\left(\mathbf{v}_{i+\frac{1}{2},j}\rho_{i+\frac{1}{2},j}-\mathbf{v}_{i-\frac{1}{2},j}\rho_{i-\frac{1}{2},j}\right)+\frac{1}{\Delta y}\left(\mathbf{v}_{i,j+\frac{1}{2}}\rho_{i,j+\frac{1}{2}}-\mathbf{v}_{i,j-\frac{1}{2}}\rho_{i,j-\frac{1}{2}}\right)=D\nabla^2\rho_{i,j}+S_{i,j}. \tag{9}$$

Similarly, the QUICK scheme is applied to the convective part of the velocity equation, ensuring accurate representation of momentum transfer under non-linear traffic interactions:

$$\frac{\Delta\mathbf{v}_{i,j}}{\Delta t}+\frac{1}{\Delta x}\left(\mathbf{v}^2_{i+\frac{1}{2},j}-\mathbf{v}^2_{i-\frac{1}{2},j}\right)+\frac{1}{\Delta y}\left(\mathbf{v}^2_{i,j+\frac{1}{2}}-\mathbf{v}^2_{i,j-\frac{1}{2}}\right)=-\frac{1}{\rho_{i,j}}(p_{i+1,j}-p_{i,j})+\nu\nabla^2\mathbf{v}_{i,j}+k_{\text{obs}}\frac{\mathbf{r}_{i,j}}{\|\mathbf{r}_{i,j}\|}. \tag{10}$$

Here, $D$ denotes the diffusion coefficient (driver heterogeneity), $\nu$ is the kinematic viscosity (traffic dissipation), $S_{i,j}$ represents source terms (vehicle entry/exit), and $k_{\text{obs}}$ models obstacle-induced deceleration near roundabouts.

### B. Time Integration with Crank–Nicolson Scheme

For temporal discretization, the Crank–Nicolson method is employed, which blends explicit and implicit formulations to achieve second-order accuracy while maintaining numerical stability. For the density equation, the semi-discrete form is:

$$\begin{aligned}&\frac{\rho^{n+1}_{i,j}-\rho^n_{i,j}}{\Delta t}+\frac{1}{2\Delta x}\left[\mathbf{v}_{i+\frac{1}{2},j}(\rho^{n+1}_{i+\frac{1}{2},j}+\rho^n_{i+\frac{1}{2},j})-\mathbf{v}_{i-\frac{1}{2},j}(\rho^{n+1}_{i-\frac{1}{2},j}+\rho^n_{i-\frac{1}{2},j})\right]\\&=\frac{1}{2}\left[D\nabla^2\rho^{n+1}_{i,j}+D\nabla^2\rho^n_{i,j}\right]+\frac{1}{2}\left(S^{n+1}_{i,j}+S^n_{i,j}\right).\end{aligned} \tag{11}$$

The velocity equation is similarly advanced in time using Crank–Nicolson, capturing the feedback between circulating and incoming flows at roundabout entries.

### C. Non-dimensionalization

To generalize the governing equations, key variables are expressed in non-dimensional form (Table II). Scaling is based on characteristic density $\rho_0$, velocity $U$, and length $L$.

**TABLE II: NON-DIMENSIONALIZATION OF GOVERNING VARIABLES.**

| Variable | Non-dimensional form |
|---|---|
| Density | $\rho=\rho_0\tilde{\rho}$ |
| Velocity | $\mathbf{v}=U\tilde{\mathbf{v}}$ |
| Length | $x=L\tilde{x},\ y=L\tilde{y}$ |
| Time | $t=\frac{L}{U}\tilde{t}$ |
| Pressure | $p=\rho_0U^2\tilde{p}$ |
| Reynolds number | $\text{Re}=\frac{UL}{\nu}$ |
| Strouhal number | $\text{St}=\frac{U}{S_0L}$ |

Substituting these variables into the discretized system yields the proposed non-dimensional governing equation:

$$\begin{aligned}&\frac{\tilde{\rho}^{n+1}_{i,j}-\tilde{\rho}^n_{i,j}}{\Delta\tilde{t}}+\frac{\tilde{\mathbf{v}}^{n+1}_{i,j}-\tilde{\mathbf{v}}^n_{i,j}}{\Delta\tilde{t}}+\frac{1}{2\Delta\tilde{x}}\left[\tilde{\mathbf{v}}_{i+\frac{1}{2},j}(\tilde{\rho}^{n+1}_{i+\frac{1}{2},j}+\tilde{\rho}^n_{i+\frac{1}{2},j})-\tilde{\mathbf{v}}_{i-\frac{1}{2},j}(\tilde{\rho}^{n+1}_{i-\frac{1}{2},j}+\tilde{\rho}^n_{i-\frac{1}{2},j})\right]\\&=\frac{1}{2\text{Re}}(\nabla^2\tilde{\mathbf{v}}^{n+1}_{i,j}+\nabla^2\tilde{\mathbf{v}}^n_{i,j})+\frac{\text{St}}{2}(\tilde{S}^{n+1}_{i,j}+\tilde{S}^n_{i,j})-\frac{1}{2\tilde{\rho}_{i,j}}(\tilde{p}_{i+1,j}-\tilde{p}_{i,j})+\frac{k_{\text{obs}}}{2}\frac{\tilde{\mathbf{r}}^{n+1}_{i,j}+\tilde{\mathbf{r}}^n_{i,j}}{\|\tilde{\mathbf{r}}_{i,j}\|}.\end{aligned} \tag{12}$$

### D. Boundary Conditions

The system is closed with boundary conditions reflecting vehicle inflow, yield-based merging, and circulation dynamics at the roundabout:

$$v(x,y,t)>0,\qquad v(x,y,0)=0,\qquad \Delta x>\frac{1}{2}. \tag{13}$$

These conditions ensure realistic representation of vehicles entering and exiting the roundabout while capturing capacity drops and delay propagation. This methodology integrates high-order spatial discretization (QUICK), unconditionally stable time-stepping (Crank–Nicolson), and non-dimensional scaling to provide a robust numerical framework for analyzing traffic flow around roundabouts.

## IV. RESULTS AND DISCUSSION

### A. Parameter Estimation and Fitting

A crucial step in validating the proposed traffic flow model is the estimation and fitting of parameters that govern the continuum formulation in (12). Parameters such as the diffusion coefficient $D$, the source term $S$, and the empirical relationships between density $\rho$, speed $V$, and flow $Q$ must be calibrated to ensure consistency with both theoretical expectations and simulated outcomes. In this study, parameter estimation was performed by extracting simulation data at probe locations and across the computational grid at the final time step. The parameter estimation and fitting process thus served two purposes: (i) validating that the numerical outputs align with classical fundamental diagrams documented in empirical studies, and (ii) providing a calibrated set of macroscopic relations that can be used for further analytical or policy-based evaluations.

**TABLE III: OPTIMAL VALUES OF PARAMETERS USED**

| Parameter | Description | Value Range | Value used | Source |
|---|---|---|---|---|
| $D$ | Diffusion coefficient | - | 0.1 | [16] |
| dt | Time step size | - | 0.01 | Assumed |
| dx | Spatial x-step size $> 1/2$ | - | 1 | Assumed |
| dy | Spatial y-step size $> 1/2$ | - | 1 | Assumed |
| $t_{final}$ | Simulation time | - | 1 | Assumed |
| S | Source term for disruptions | - | 0.05 | Assumed |

### B. Numerical Simulation and Discussion

Fig. 2 presents the simulated fundamental relations between speed, density, flow, and area occupancy. The quadratic fits confirm well-known traffic theory outcomes: mean speed initially increases with occupancy but declines as density builds up, leading to an inverted parabola. Flow increases with density up to a critical point (capacity) before collapsing under congested conditions. This mirrors the empirical studies that consistently report concave speed–density and flow–density relations, with a maximum flow at an intermediate density [3, 6, 4]. The recovery of these patterns from (12) highlights the validity of the finite-volume discretization and its alignment with classical continuum models such as LWR and Payne–Whitham.

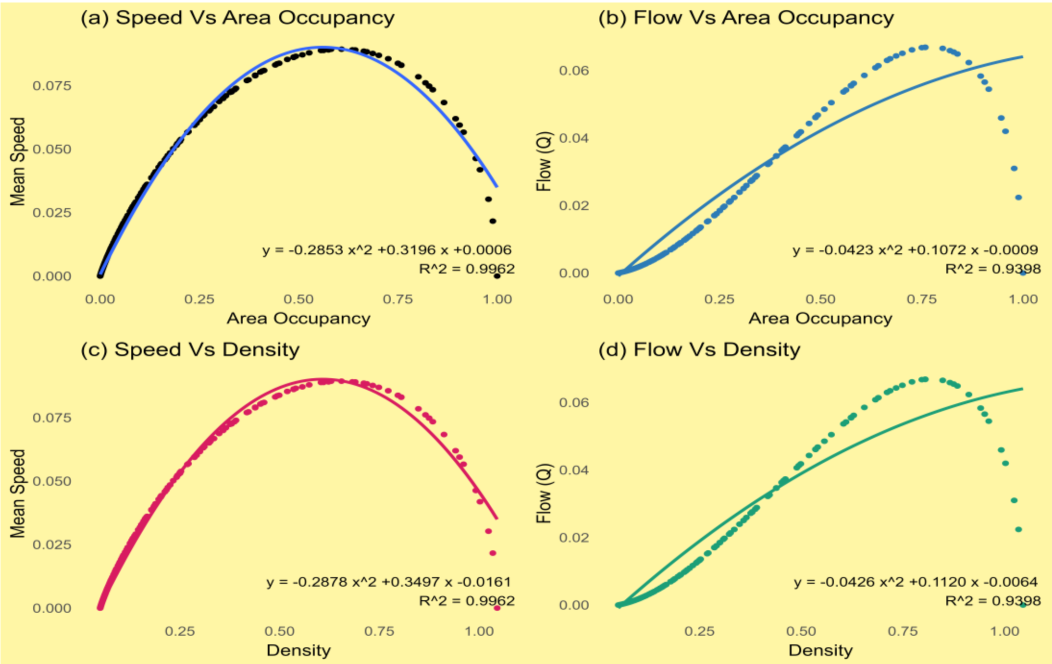


**Fig. 2: Fitted quadratic relations from (12) simulation: (a) Speed vs Area Occupancy, (b) Flow vs Area Occupancy, (c) Speed vs Density, (d) Flow vs Density.**

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**1) Scenario Comparison**

Fig. 3 compares two diffusion scenarios ($D = 0.1$ and $D = 0.2$) in terms of normalized throughput. Scenario A (lower diffusion) achieves higher throughput, while Scenario B (higher diffusion) dampens density gradients, reducing flow but improving stability. From a mathematical perspective, the diffusion coefficient in (12) regulates the balance between advection and dissipation: a higher $D$ suppresses oscillations but reduces gradient-driven acceleration, leading to lower mean flow. This trade-off is consistent with empirical findings where ramp metering, signal control, or roundabout priority rules stabilize flow but reduce achievable capacity [12, 10]. Hence, the numerical simulation provides a mechanistic explanation for observed policy–capacity trade-offs in real networks.

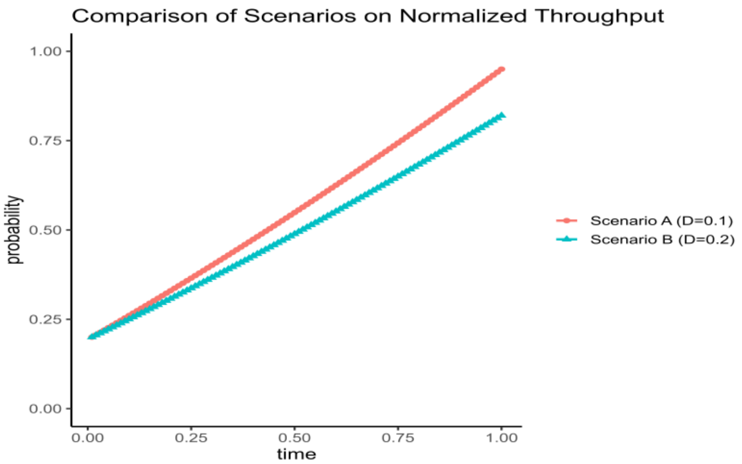


**Fig. 3: Comparison of normalized throughput trajectories under two diffusion coefficients.**

**2) Temporal Probe Diagnostics**

Fig. 4 shows time–series diagnostics from a probe location. The time–headway plot indicates a decline from unstable high values to a stabilized range. Time–density and time–speed are inversely related, as expected, showing oscillations linked to transient source terms. The speed–density scatter, colored by headway, illustrates that shorter headways coincide with moderate densities where speeds remain non-zero. At very high densities, both speed and flow collapse. These oscillatory behaviors are directly traceable to the advection–diffusion interplay in (12), where the advective term produces propagating density waves and the diffusion term smooths them out. Such dynamics are consistent with empirical detector-based observations of stop-and-go waves and spillback near roundabouts [11, 2].

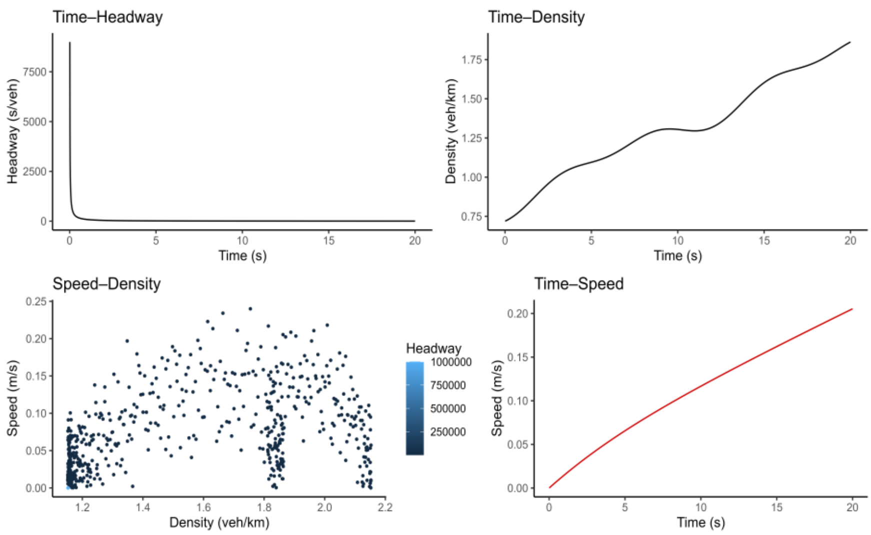


**Fig. 4: Time–series diagnostics at a probe location: (a) Time–Headway, (b) Time–Density, (c) Speed–Density (colored by headway), (d) Time–Speed.**

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**3) Discussion**

The simulation results validate the continuum formulation in (12), which successfully reproduces fundamental diagrams, throughput trade-offs, and probe-based oscillations. This aligns with [6, 4], who highlighted that while macroscopic and kinetic models capture shock waves and queues, most intersection and roundabout studies still rely on statistical gap-acceptance methods [10]. By explicitly recovering an interior maximum in flow (Fig. 2), the model demonstrates its ability to capture capacity phenomena observed in field data. The scenario comparison (Fig. 3) mirrors evidence that greater control increases stability but reduces effective capacity. Likewise, the time–series analysis (Fig. 4) reveals oscillatory stop-and-go dynamics consistent with empirical observations of congested roundabouts [11, 5].

These results underscore three implications:

1. The optimal density for maximum throughput can be explicitly computed from the PDE framework rather than estimated statistically, strengthening predictive capability;

2. There exists a measurable trade-off between stability (via diffusion) and flow efficiency, reflecting real-world signalized and priority-based control effects;

3. Embedding roundabout-specific interface laws into this PDE system will allow explicit modeling of capacity drop, spillback, and queue interaction with upstream signals, thereby addressing limitations raised in the empirical literature.

## V. CONCLUSION

Traffic congestion around roundabouts remains a major challenge in urban transport systems, with significant socio-economic and environmental implications. While roundabouts can improve capacity and safety, they also introduce turbulence, queue spillback, and localized bottlenecks if poorly designed. Existing macroscopic and kinetic traffic models often fail to represent roundabout-specific dynamics, particularly yield priority, circulating feedback, and capacity drops, within a rigorous mathematical framework. This limitation hinders the ability to predict congestion patterns and evaluate policy interventions in complex networks.

The proposed Navier–Stokes–advection–diffusion framework successfully reproduces fundamental traffic relations, including concave speed–density and flow–density curves with identifiable capacity points. Simulation results demonstrate that:

1. Optimal density and throughput can be derived directly from the PDE model.

2. Diffusion parameters quantify the trade-off between stability and flow efficiency.

3. Temporal probe diagnostics reveal oscillatory stop-and-go dynamics consistent with empirical observations.

These findings validate the mathematical model's capability to capture realistic traffic phenomena at roundabouts. By embedding priority laws into continuum equations, the model provides a unified approach that bridges theoretical rigor and empirical relevance, addressing a longstanding gap in traffic flow modelling. For policy, the results underscore the importance of designing roundabouts with calibrated entry–circulation interfaces to balance stability and efficiency. Future research should extend the model by:

1. Incorporating stochastic demand processes to reflect bursty, dependent arrivals.

2. Calibrating pressure, viscosity, and diffusion functions using empirical sensor and video data from real roundabouts.

3. Embedding adaptive control strategies (e.g., dynamic metering) to test policy scenarios within the PDE framework.

Such extensions will enhance predictive accuracy and support sustainable urban congestion management.

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